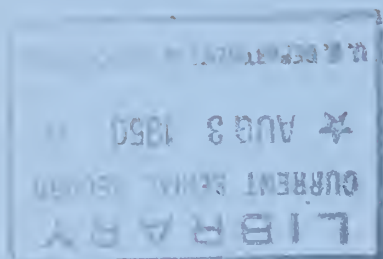


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GROWTH AND YIELD OF UPLAND BALSAM FIR IN THE LAKE STATES

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INTRODUCTION

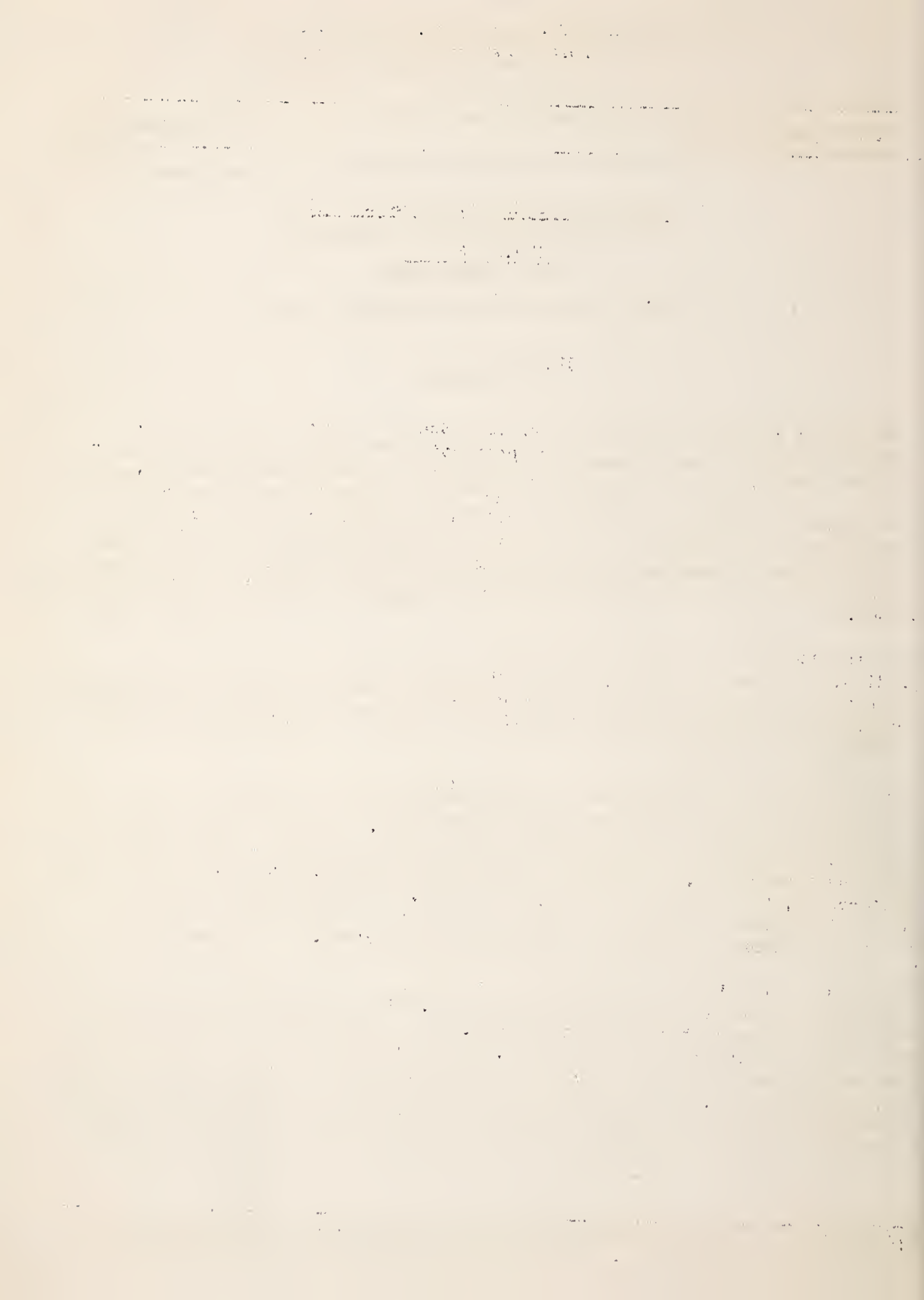
Balsam fir has been increasing in the Lake States during recent decades. Mainly responsible are improved fire protection, the species' ability to reproduce readily, and its inherent tolerance or capacity to endure shade. Since it is a valuable pulpwood and since the supply of spruce pulpwood has been rapidly declining, a demand has arisen for knowledge on the growth and development of balsam fir. To meet this need the Lake States Forest Experiment Station, in cooperation with various industrial, state, and federal groups, during 1948 collected growth information on balsam fir over the three Lake States.

From the analysis of these data there has been developed a set of tables for predicting growth of balsam fir as it occurs in stands of different densities and mixtures with other species on upland sites (see tabular section). The accompanying text is intended primarily to explain the application of the tables in the field.

This study has been restricted to upland balsam fir undisturbed by very recent cuttings or fire. It includes stands in narrow belts around swamps or on moist upland mineral soils but not true bog sites. Although found occasionally in almost pure stands, balsam fir occurs more frequently as a component of such types as aspen-birch, spruce-balsam, northern hardwood, ash-elm, northern white-cedar, and occasionally hemlock or pine. In these stands balsam fir seldom is strictly even-aged. Usually there are remnants of former stands and an understory of recently established reproduction.

When mixed with other species balsam fir encounters varying degrees of competition which affect its growth and development. It may occur in clusters or scattered uniformly throughout the stand. The density of such stands may again vary considerably from place to place. Therefore, in evaluating growth allowance must be made for the diversity of conditions such as varying composition, density, age, site, and competition with other species. The problem is more complicated than that usually encountered in yield table studies and requires a somewhat different approach.

1/ Maintained at University Farm, St. Paul, Minnesota, in cooperation with the University of Minnesota.



FACTORS AFFECTING GROWTH

The stand characteristics which are needed for evaluation of gross growth are stand age, degree of competition, site quality, and proportion of balsam fir. A description of each of these factors is given below, followed by a detailed explanation of the field procedure.

Stand Age

Age, as used in this study, is that taken at breast height. This age was found to be less variable than the total age and showed better correlation with the observed growth. Furthermore, butt rot frequently interferes with the measurement of total age, especially in the older trees.

In most of the mixed stands, trees with ages ranging from 85 to 115 percent of the average stand age contain about 62 percent of the total stand basal area. The remaining 38 percent of the basal area is equally divided between trees less than 85 percent and more than 115 percent of the average age. In other words, there is a main stand, with ages fairly closely distributed around the average, plus younger individuals and occasional trees from former stands.

For convenience, only the age of dominant and codominant trees in the main stand was used. To arrive at the average stand age, the breast-height ages of these trees in different diameter groups were averaged using basal areas as rough weights. If the balsam fir is essentially even-aged, no weighting is needed.

Competition Index

Description of Competition

The degree of density and the amount of overhead competition, especially that exerted by other species, are very important factors affecting the development of balsam fir. These factors vary considerably from stand to stand and are among the most important stand characteristics. As a typical illustration of these characteristics, a tally of three different stands, each 45 years old and on similar sites, is presented in the following tabulation:

	Number of trees by diameter groups			Total basal area	percent of	
	1 to 5 inches	6"+	All	Square feet	percent of an assumed normal	balsam fir basal area other than suppressed
<u>Stand #1-Low competition</u>						
Balsam fir						
Suppressed.....	55	1	56	2.6
Total.....	<u>143</u>	<u>87</u>	<u>230</u>	<u>39.1</u>	<u>...</u>	<u>93</u>
All species.....	263	155	418	71.4	45	..

<u>Stand #2-Medium competition</u>						
Balsam fir						
Suppressed.....	275	1	276	10.8
Total.....	470	179	649	79.0	...	86
All species.....	545	236	781	112.1	74	..

<u>Stand #3-High competition</u>						
Balsam fir						
Suppressed.....	520	8	528	15.1
Total.....	827	193	1,020	87.7	...	83
All species.....	1,131	311	1,442	150.6	105	..

It is observed that in the low competition group the density in terms of total basal area is less than half that of the high competition group, and contains only one-sixth as much suppression in balsam fir. Low density and a low degree of suppression, therefore, go hand in hand in defining competition in a stand.

The effect of overhead competition is also apparent. The high competition stand contains twice as many trees 6 or more inches d.b.h. as the low competition group, but ten times as many suppressed balsam firs. Overhead competition, therefore, as well as the density, affects the amount of suppression. In the example above, the percentages of suppressed balsam fir (basal area) for the low, medium, and high competition groups are 7, 14, and 17 percent respectively.

Evaluation of Competition

In order to measure the combined effect on growth of the degree of density and the amount of competition offered by larger trees, a factor called the competition index was devised. This is expressed as a ratio: $\frac{d}{P} = C$, where d

is the basal area of the entire stand as a percent of the assumed normal (table 4), and P is the percent of the balsam fir basal area in trees other than suppressed. In the preceding example, these ratios and their combined effects line up in the following manner:

	Percent of assumed normal basal area (d)	Basal area percent of balsam fir trees other than suppressed (P)	Competition index (C)
Stand #1-Low competition	45	93	.48
Stand #2-Medium competition	74	86	.86
Stand #3-High competition	105	83	1.27

[illegible][illegible]

The assumed normal basal area per acre of all species in the stand is shown in relation to average stand diameter (table 4 in tabular section). The degree of stocking, therefore, can be determined by dividing the actual stand by the "normal" basal area.

The amount of crowding as portrayed by the competition index naturally affects the volume and growth of balsam fir. Generally, for any diameter both the form factor and the total height of all trees for any given age and site increase with the competition index. In fact the difference in volume per tree for any d.b.h. between the lowest and highest competition index groups is as much as 17 percent. However, under similar age and site conditions the average diameter of balsam fir is greater under low competition and smaller under higher competition.

Because of low density, the volume growth per acre of stands having a low competition index is actually less than that of stands having a high competition but the rate of growth is appreciably higher due to less crowding both from the sides and above. Conversely, the stands of high competition have a larger total growth per acre but a smaller rate of growth than less dense stands.

Change in Competition

Degree of competition does not remain the same^{2/} even within a relatively short period of time, such as one decade, except in very old stands or in stands where the density is already near the assumed normal. The density of understocked stands increases with age, whereas that of overstocked stands declines. Both, in other words, tend to approach normality. The proportion of trees other than suppressed, in turn, tends to increase with age in all types of stands. The change in density of stocking and the proportion of more vigorous trees are reflected in the change in competition index (table 1).

It is clear that the greatest change in competition index occurs in young stands especially those which deviate considerably from normal stocking which is approximately at competition index of 1.2 (table 1).

To predict growth, therefore, it is necessary to make allowance for these anticipated changes in the next 10 and 20 years (see tables 7 to 12, tabular section).

^{2/} Tables 13, 14, and 15 (see tabular section) show volume per acre in total cubic feet and in cords in relation to age, site, and competition index. They should not be used for estimating growth since they assume no change in competition index as the stands grow older.

Table 1.--Estimated change in competition index in 10 years

Present stand age	Competition index 10 years hence when present competition index is:						
	.40	.60	.80	1.00	1.20	1.40	1.60
<u>Years</u>							
10	.55	.76	.90	1.06	1.20	1.25	1.35
20	.49	.70	.87	1.03	1.20	1.29	1.41
30	.46	.67	.84	1.01	1.20	1.30	1.44
40	.44	.64	.82	1.00	1.20	1.32	1.47
50	.42	.62	.81	1.00	1.20	1.33	1.48
60	.41	.61	.80	1.00	1.20	1.34	1.49
70	.40	.60	.80	1.00	1.20	1.34	1.50

Site Quality

Site quality is the relative productive capacity of the land. Usually it is determined from the size of timber in relation to its age. This is illustrated in the example below which shows three stands all having the same competition index and all being exactly 45 years of age:

Characteristics of average dominant and codominant trees	Relative development of dominant and codominant trees		
	Poor	Medium	Good
D.b.h. (inches).....	6.0	7.6	9.0
Annual growth in d.b.h. in the last 20 years (inches)	.11	.15	.19
Total height (feet).....	40	50	54
Volume per tree (cubic feet)	3.1	6.3	9.6

Although these stands are similar in regard to age and competition, they differ considerably in the rate of development of dominant and codominant trees. The most obvious explanation is that the soil and other ecological factors differ in their natural potential or productivity as indicated by such stand characteristics as average d.b.h., height, rate of growth, and volume.

Conventionally, site is measured by the height of average dominant and codominant trees as related to age in relatively pure stands and under normal conditions of stocking. However, since these conditions are the exception rather than the rule in upland balsam fir, it was found that the average volume per tree as it relates to age and the degree of competition is a better index of site than height of dominant trees.

It is apparent in the example above that the excellent development of the third stand in d.b.h. and rate of growth is not as well explained by the height of dominant trees as by the average volume per tree. The site quality of this stand, if judged by height alone, would have been determined as only medium. Yet this stand, only four feet taller than the second stand, showed approximately 50 percent more volume per tree and about a 25-percent higher rate of diameter growth. The dominant trees within each stand showed an even rate of d.b.h. growth for the past 40 years indicating that there was no cutting or other disturbance in any of the three stands.

Quite commonly height failed to evaluate site quality as well as the average volume. Consequently, the average volume of dominants and codominants (which reflects the effects of site on both height and d.b.h.) was selected as a measure of site quality.

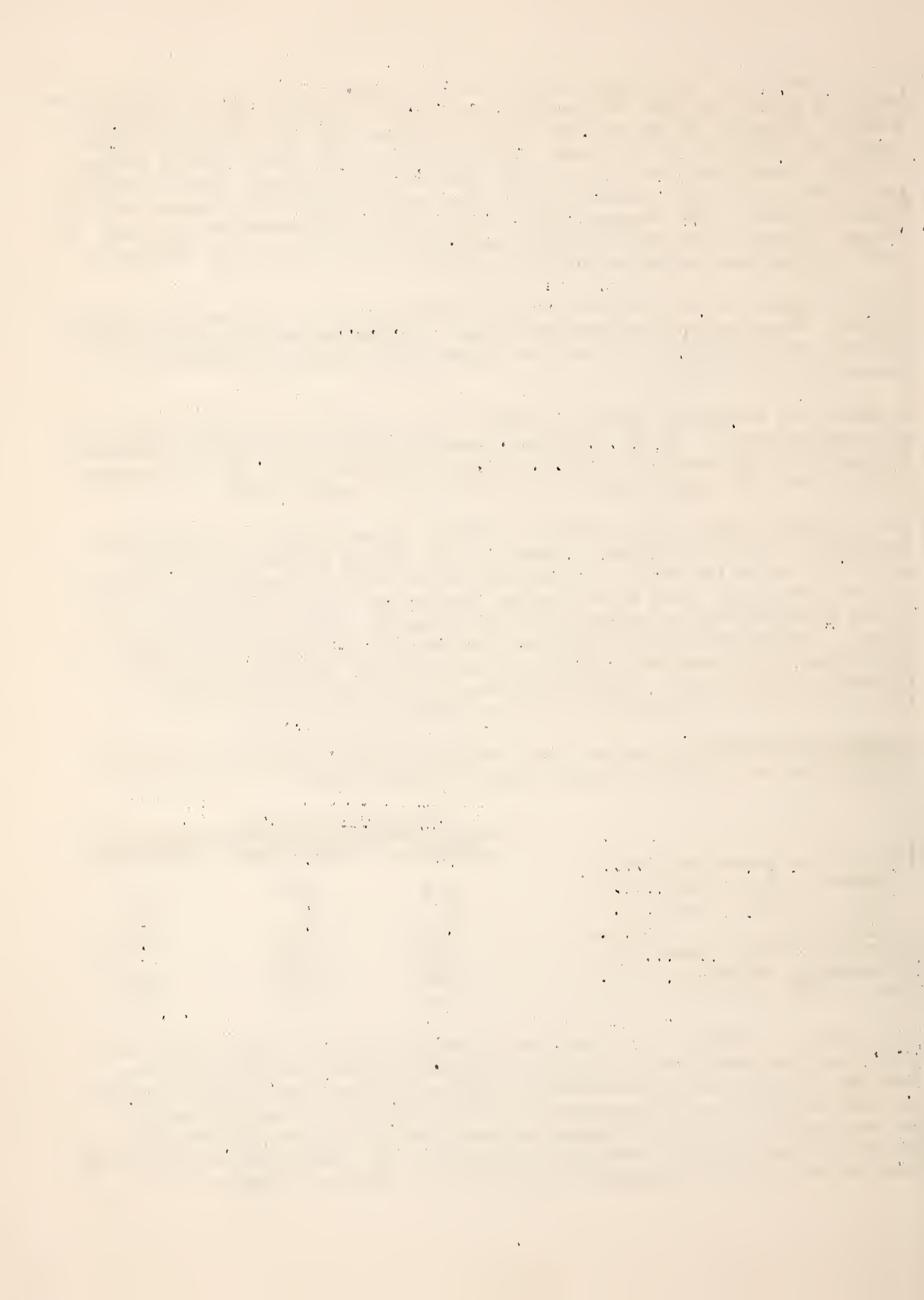
A simple and convenient index of volume is the product of average basal area and average height. In the foregoing example these volume indexes (hereafter designated as vi) are 7.8, 15.7, and 23.9, which line up in the same proportion as the true volume scale of 3.1, 6.3, and 9.6 cubic feet.

These vi values were curved over age and the entire range of variation was separated into five arbitrary groups of site quality (table 5 in tabular section). Since the trend corresponded to the average competition index of .8, the latter was taken as a standard for average volume development. For stands with competition indexes above or below .8, some correction is required. As described previously, the average size of trees in the stand diminishes with increased competition. Accordingly, vi values should be adjusted to the average competition index by means of certain factors (table 6) before site quality is finally determined (table 5).

By way of illustration, assume three stands 40 years of age--one each with low, average, and high competition index as shown below:

	Competition index		
	Low (.4)	Average (.8)	High (1.2)
Dominant and codominant trees:			
Average d.b.h. (inches)....	8.8	7.0	6.0
Average height (feet).....	40	45	50
<u>vi</u> values.....	16.9	12.0	9.8
Correction factor (table 6)...	1.17	1.00	.84
Corrected <u>vi</u> values.....	14.4	12.0	11.7
Site quality (table 5).....	Good	Fair	Fair

The corrected volume index (basal area x height) values of 14.4, 12.0, and 11.7, indicate good site for the low competition stand, and fair site for both the average and high competition stands. Had the original vi value of 9.8 for the high competition stand not been corrected to 11.7, the site quality would have been determined as poor instead of fair (see table 5). The correction factor of .84 simply allowed for the depressing effect of high competition on the development of dominant and codominant trees.



In some cases where there are a few large trees, obviously remnants of a former stand rather than a part of the main stand, a more accurate estimate of site index would be obtained by the elimination of such trees from the calculation of vi.

No attempt has been made to appraise site quality on any basis other than volume index. This index, of course, should correlate with the combination of factors affecting tree growth such as soil quality, soil moisture, drainage, etc. It has been observed in this study that the volume index of balsam fir associated with northern hardwoods and ash-elm averages considerably higher than in mixture with aspen-birch or northern white-cedar. The lowest average volume index was found when balsam fir occurred in mixture with black spruce or on sandy sites.

All site indexes based on the rate of development of dominant trees either in height or volume, assume no sudden changes in the stands such as release due to cutting or other causes. There is always some lag in adjustment of stand development to any drastic change. Under these conditions the site index representing the growth potential is more likely to be underestimated. As a guide it is recommended that all borings obtained for estimating breast-height age be carefully examined for evidence of release or past suppression.

Proportion of Balsam Fir

The volume and growth tables presented in this report assume 100 percent balsam fir. The balsam fir component of the stand (basal area and volume) was inflated to a 100-percent basis, assuming the same development of the entire stand as that of the balsam fir portion alone. Consequently, in mixed stands reduction must be made depending on the actual amount of balsam fir present. Considerable variation occurs among stands, but since there does not seem to be any relation to age, it was assumed that the proportion will not change in the next 10 and possibly 20 years. The reduction in growth values, therefore, can be made on the basis of the present basal area. No attempt was made to evaluate the growth of the associated species.

The proportion of balsam fir does not appear to be correlated with site but, as would be expected, there is some correlation with competition index. Generally, the amount of balsam fir decreases with an increase in competition index, varying on the average from 62 percent for low competition to 37 percent for high competition.

FIELD PROCEDURE AND APPLICATION OF TABLES

most

These growth tables probably will be/useful in appraising timberland for purchase or sale, or in connection with management problems on selected tracts. Estimates usually will be made by means of small sample plots scattered throughout the tract, the number depending on the accuracy required. For areas which are classified by forest conditions or are divided into smaller tracts, each subdivision will require a separate estimate. For rough estimation where only a general picture of volume growth capacity is required, a reconnaissance employing ocular estimates of the growth factors may be made without the use of sample plots.

The factors needed for application of growth tables can be obtained from the sample-plot measurements as follows:

1. On each plot, tally by d.b.h. classes (1) all the balsam fir trees by three crown classes, dominant and codominant, intermediate, and suppressed; and (2) all other species combined. The tally should exclude trees more than 60 percent defective (cull) and dead trees.
2. Compute the basal areas for each of the groups listed above. A multiple basal area table (table 16) is included to simplify the calculation of these figures.
3. Determine the degree of stocking of all species in the stand. This is the ratio of the total basal area to the basal area assumed to be normal (table 4).
4. Determine the proportion of balsam fir which is not in suppressed trees, by dividing the basal area in dominant, codominant, and intermediate trees by that of all balsam fir. With sufficient experience this can be estimated by eye.
5. Compute the competition index by dividing the stocking ratio (step 3) by the ratio obtained in step 4.
6. At breast height, bore a sufficient number of dominant and codominant balsam fir trees to represent different diameter groups of the main stand. The average age at breast height of all diameter groups can then be obtained, using basal areas of the different groups as rough weights.
7. Calculate the average diameter of dominant and codominant balsam fir trees, using basal areas recorded previously.
8. Measure total height on bored trees and on a few additional trees. Compute average height, using number of trees as weights. Experienced men probably can estimate the average height corresponding to the trees of average d.b.h.
9. As explained previously, site index is determined by means of the product of the basal area of the average dominant and codominant balsam fir tree and its height (steps 7 and 8). These products, called yi, are presented in a site index table (table 5), for competition index 0.8. If the competition index is other than 0.8, yi should be divided by appropriate correction factors (table 6) before the site-index table can be used.

10. The last factor needed is the proportion of balsam fir. This is the ratio of the basal area of balsam fir to the total basal area in the stand. This factor can also be estimated by eye.
11. After the growth factors have been determined, the expected growth for the next 10 and 20 years can be calculated from values given in growth tables (tables 7 through 12). Example:

Age	40 years
Competition index	.6
Site index	Fair
Proportion of balsam fir	.36 (36 percent)
Expected 10-year growth per acre:	
Cubic feet (table 7) = $400 \times .36 = 144$ cubic feet, gross,	
Cords to 3-inch top (table 9) = $5.4 \times .36 = 1.9$ cords, gross.	
Cords to 4-inch top (table 11) = $5.2 \times .36 = 1.9$ cords, gross.	
Expected 20-year growth per acre:	
Cubic feet (table 8) = $640 \times .36 = 230$ cubic feet, gross.	
Cords to 3-inch top (table 10) = $9.2 \times .36 = 3.3$ cords, gross.	
Cords to 4-inch top (table 12) = $3.8 \times .36 = 3.2$ cords, gross.	

The estimated growth of 0.19 cord annually, which appears small, actually represents a rather substantial rate of increase considering the fact that the site is only fair and the stand in question contains only about 4 cords of merchantable balsam fir per acre.

Another point that may require clarification is the similarity of growth in cords when merchantability is considered both to a 3-inch and 4-inch top. The cordwood growth to these different top utilizations is about the same at ages near 40 to 50 years. Generally, in young stands (below 40 years of age) the cordwood growth to a 3-inch top is considerably higher than that to a 4-inch top. The reverse is true of older stands. These changes are caused by different rates of ingrowth from smaller trees as well as differences in growth of tops.

The rate of growth computed above assumes basic volumes as given in tables 13 to 15. If the actual stand volume has been determined by means of local volume tables and it was found to either underrun or overrun the basic tables by a given percentage the growth can be either reduced or increased by the same percentage.

DEFECT AND NET GROWTH

Defect

The previous sections described the evaluation of gross growth. In most cases, however, the chief concern is the estimation of net increment of merchantable wood. The gross growth, therefore, must be reduced for defect and cull losses.

The amount of defect varies greatly among stands over a relatively small area. It increases with age, the rate of increase depending on the amount of defect present. The higher the percent defect the greater the cull losses in succeeding years. Red rot in the upper bole and butt rot are important sources of defect, especially in the older stands. In mature and overmature stands rots progress very rapidly. They cause excessive mortality through wind breakage and frequently result in negative net yields. In predicting future yields of stands over 50 years of age it is important, therefore, to observe closely the amount and type of defect present.

On the basis of data collected in this study and the information obtained by the Forest Survey in the Lake States, the following average trend of defect with age has been observed:

Table 2.--Average percent of defect in merchantable trees in relation to age

Stand age at breast height	Average percent of defect 1/	Stand age at breast height	Average percent of defect 1/
10	.5	60	8.0
15	.9	65	9.2
20	1.4	70	10.3
25	2.0	75	11.5
30	2.6	80	12.8
35	3.3	85	14.0
40	4.1	90	15.4
45	5.0	95	16.7
50	6.0	100	18.0
55	7.0		

1/ Culled material not utilizable for pulpwood.

The percent defect shown is for merchantable trees only and does not consider cull trees or those less than 40 percent sound. The loss from such trees has been accounted for in the yield tables where it becomes noticeable after 70 years of age, the mortality and cull losses resulting in a downward trend in the yields per acre. Accordingly, the cull loss from these trees, although large, need not be considered here.

The trend of defect in the above table represents averages for a large number of stands, and is not strictly applicable to any individual stand. Consequently the actual cull percent should be estimated in the field for each stand. The trend values can be used to determine the rate of increase in defect of merchantable trees over any given period. The assumption is then made that the same rate of increase in cull percent over age will apply to the actual present defect as determined in the field.

For example, the average defect shown for 50- and 60-year-old stands is 6 and 8 percent respectively. This represents an increase in 10 years of 33 percent. If the actual cull percent obtained for the 50-year-old stand is 9 instead of 6, it is assumed that the increase in cull loss in 10 years will be 33 percent of 9 or 3 percent. Thus at 60 years of age the stand is assumed to have approximately 12 percent defect. This proportionality, therefore, allows greater cull loss in volume for stands which are found to be more than normally defective.

Net Growth

As stated previously, allowances were made in the yield tables (tables 13, 14, and 15) for average losses due to mortality and wholly defective trees, but not for losses due to defect in merchantable trees. The net growth is the difference between estimated future sound volume and the present sound volume. It can be worked out in the following manner:

Stand conditions

Age	40 years
Competition index	1.0
Site quality	Good
Proportion of balsam fir	60 percent
Percent defect	7 percent

Present stand (per acre)

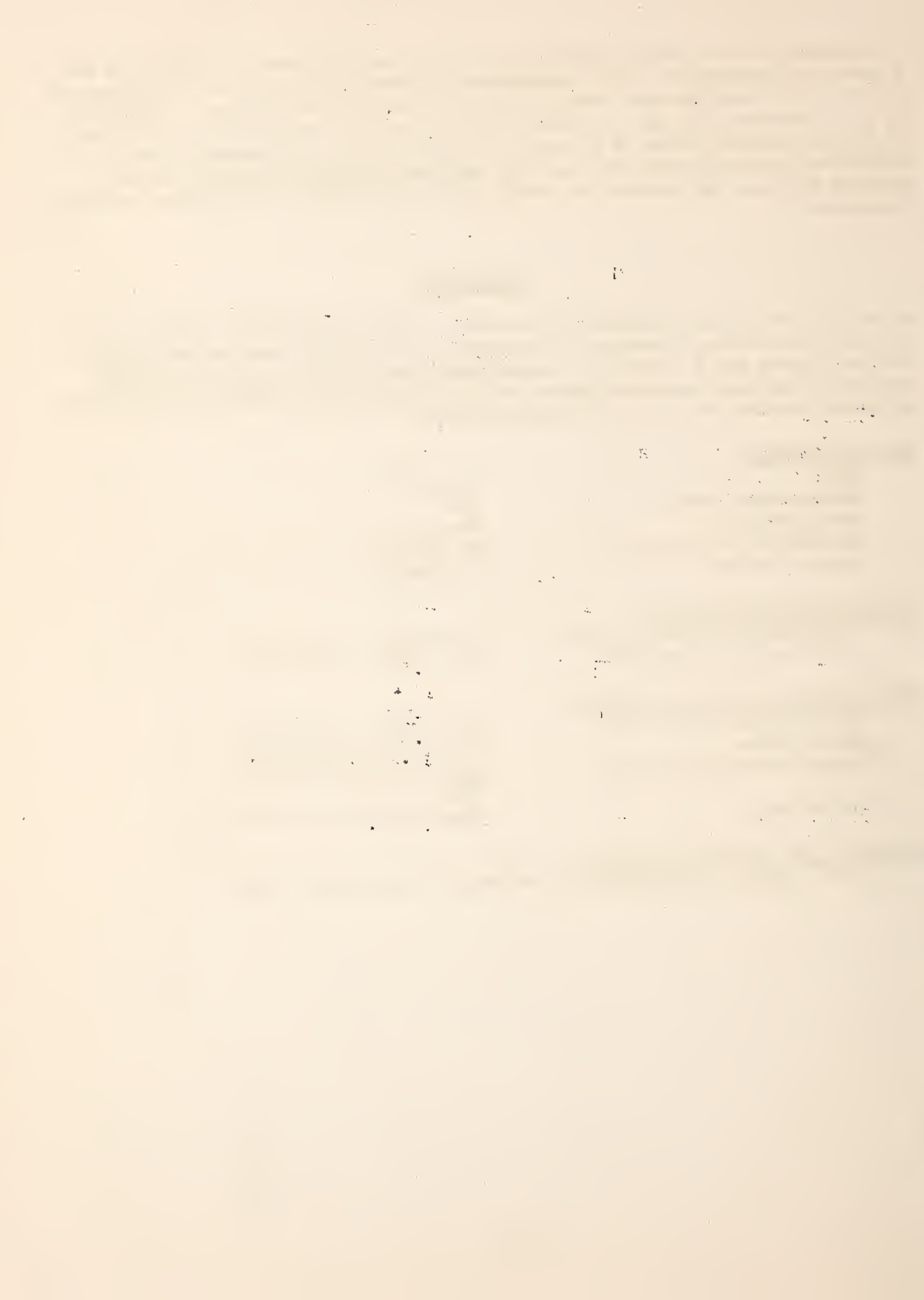
Gross volume of balsam fir	14 cords
Net volume of balsam fir	$14 \times .93 = 13.0$ cords

Stand 10 years hence (per acre)

Gross growth (table 11)	$8.1 \times .60 = 4.9$ cords
Gross volume	$14 + 4.9 = 18.9$ cords
Percent defect (table 2)	$\frac{6.0}{4.1} \times 7 = 10.2$ percent
Net volume	$18.9 \times .898 = 17.0$ cords

Estimated net growth (per acre)

$17.0 - 13.0 = 4.0$ cords in 10 years, or .40 cord per annum



ROTATION AGE

Balsam fir in the Lake States is primarily a pulpwood species. Since the quality increment is of minor consideration, mortality and cull losses are the chief factors in determining the rotation age. Barring any specific financial considerations of management, the balsam fir stands should be liquidated before they reach the pathological rotation age, i.e., immediately preceding the time when no net increment is expected. When the declining growth is offset by cull losses in merchantable trees, no net growth occurs and is immediately followed by actual loss in volume per acre.

The pathological rotation age, governed by the amount of defect present in the stand, varies with site quality and competition index. For convenience in determining this rotation age, there has been prepared a guide table (table 3) which shows the percent of present defect (merchantable trees) which, with allowance for further deterioration, will completely offset the expected gross growth during the next 10 years.

For example, if a 50-year-old stand on good site with a competition index of .6 shows a defect in merchantable trees of 22 percent, it will still acquire some net growth in the next 10 years since it has less than the 32 percent defect indicated in the table. However, it should be cut shortly after it reaches the age of 60 (21 percent cull) since no net growth will be expected after that age. A more advanced stand of 70 years of age under the same conditions could tolerate a defect of only 12 percent rather than 32 percent.

Generally, stands showing especially high defect should be cut sooner than indicated by the table since such stands are apt to show greater mortality than has been allowed for in the yield tables. In balsam fir there is always considerable risk of losing trees through windfall, breakage, etc., and this risk increases with age and defectiveness of stand. Thus a 50-year-old stand on good site with competition index .6 which is 32 percent defective should be cut immediately, even though the table indicates that no actual loss in volume is expected during the next 10 years.

Table 3.--Percent defect in merchantable trees resulting in zero net growth during the next decade 1/

POOR SITE								
Stand age : at : breast height: (years) :	Percent of defect when competition index is:							
	.4	.6	.8	1.0	1.2	1.4	1.6	
40	51	52	53	54	54	56	58	
50	44	45	47	50	53	56	59	
60	32	31	33	38	43	48	53	
70	12	15	19	22	25	32	40	
FAIR SITE								
40	42	42	43	45	48	51	54	
50	36	36	37	38	41	45	49	
60	26	25	26	28	31	35	40	
70	09	12	14	16	20	25	30	
MEDIUM SITE								
40	39	39	39	40	42	44	47	
50	34	33	33	34	36	39	41	
60	24	23	23	24	27	30	34	
70	11	13	14	15	17	20	26	
GOOD SITE								
40	36	36	35	36	37	40	42	
50	32	32	32	32	33	35	37	
60	22	21	21	21	24	27	30	
70	11	12	14	14	16	20	23	
EXCELLENT SITE								
40	35	35	34	34	34	36	38	
50	31	31	31	31	31	33	35	
60	22	21	21	21	22	24	28	
70	09	10	12	12	14	17	20	

1/ Based on table 11, giving cordwood growth to a variable top d.i.b. of not less than 4 inches.

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ACCURACY OF VOLUME AND GROWTH TABLES

When different condition classes, such as age, site, and competition index, are determined accurately, the variability of volume per acre of balsam fir within a condition is relatively small. As a matter of fact stand volume tables (tables 13 to 15) could be used for cruising timber, although this is not necessarily recommended. The variability between the actual volume per acre and that estimated from the tables is shown below in terms of coefficients of variation, by stand age and competition index. This tabulation applies to all sites:

Stand age (years)	<u>Coefficients of variation^{1/} of volume when</u> <u>competition is:</u>			
	<u>Low</u>	<u>Medium</u>	<u>High</u>	<u>All</u>
0-29	.30	.26	.37	.35
30-49	.21	.14	.18	.19
50+	.18	.14	.20	.18
All ages	.23	.15	.21	.21

^{1/}Coefficient of variation is the ratio of the variability (standard deviation) and the mean volume.

It is observed that the coefficients of variation are about the same for stands 30 years of age or older, but are considerably higher for very young stands where the average volume per acre is small. Also there is more variability in stands of either low or high competition index than in those of medium competition index. This is probably caused by the wider range in density and the proportion of dominant and codominant trees encountered under high competition. It may be assumed, therefore, that the standard error per plot for estimating volume within a specified condition class is about 20 percent of the volume itself, and that error will diminish as more plots are taken within the same condition class.

A tract of land, whether small or large, generally includes areas with different growing conditions. The most common procedure for estimating growth would be to take a series of plots throughout the tract--the intensity depending on the relative importance of the growth determinations required--on each of which age, site, and competition index would be determined. The corresponding growth estimates would then be obtained (from tables 7 to 12) and averaged for the entire tract. The accuracy of this average growth for the tract will depend not only on the sampling error resulting from the number of plots taken and the variability of the conditions involved but also on the differences between the actual and the estimated growth for each plot.

As a rough rule the percent of standard error of gross growth prediction on a tract of varying conditions can be estimated by dividing 70 by the square root of the number of plots taken. For example, with 25 plots, the standard error would be 14 percent; with 100 plots, 7 percent, etc.

This rule would tend to overestimate the error under more uniform conditions of growth and underestimate it when growth per acre is more variable than assumed by the coefficient of variation of 70, especially when stands of very slow growth predominate.

The error of the net growth predictions will, of course, be considerably higher since it also involves the error in estimating defect. No attempt has been made to estimate the possible error from this source.

In general, it is realized that the errors of growth prediction are large especially when small or heterogeneous tracts are involved. Furthermore, these errors do not take into account the possibility of disastrous losses due to windstorm, disease, insect damage, or drastic changes in weather.

Despite the uncertainties, the growth tables presented are more accurate than any estimates that even the most experienced man could make without their aid. They should be especially useful in evaluating and comparing growth possibilities of different stands.

1. The first part of the report is a general introduction to the subject of the study. It discusses the importance of the study and the objectives of the research.

2. The second part of the report is a detailed description of the methodology used in the study. It includes information about the sample size, the data collection methods, and the statistical analysis techniques.

3. The third part of the report is a discussion of the results of the study. It compares the findings with the previous research and discusses the implications of the study.

4. The fourth part of the report is a conclusion and a list of references. The conclusion summarizes the main findings of the study, and the references list the sources of information used in the study.

5. The fifth part of the report is an appendix containing additional information related to the study, such as raw data, questionnaires, and interview transcripts.

6. The sixth part of the report is a bibliography listing the sources of information used in the study.

7. The seventh part of the report is a list of abbreviations and a glossary of terms used in the study.

8. The eighth part of the report is a list of figures and tables used in the study.

9. The ninth part of the report is a list of footnotes and a list of references.

10. The tenth part of the report is a list of appendices containing additional information related to the study.

11. The eleventh part of the report is a list of references.

12. The twelfth part of the report is a list of references.

Table 4.--Assumed normal basal area stocking (all species) in relation to average stand diameter, upland balsam fir type

Average d.b.h. of : all species :	Basal area per acre	:	Average d.b.h. of : all species :	Basal area per acre
<u>Inches</u>	<u>Sq. ft.</u>	:	<u>Inches</u>	<u>Sq. ft.</u>
2	84	:	7	169
3	117	:	8	174
4	137	:	9	177
5	151	:	10	179
6	161	:	12	180

Table 5.--Site quality index based on volume index of average dominant and codominant tree, upland balsam fir type

Stand age: at breast: height (years) ^{2/}	Limits of volume index ^{1/} values when site quality is --													
	Poor		:	Fair		:	Medium		:	Good		:	Excellent	
10	Less	than	.2	.2			.3			.4			.5 or more	
20	"	"	1.6	1.6- 2.5			2.6- 3.2			3.3- 4.4			4.5 " "	
30	"	"	3.5	3.5- 5.6			5.7- 7.4			7.5- 9.9			10.0 " "	
40	"	"	6.3	6.3-10.0			10.1-13.2			13.3-17.6			17.7 " "	
50	"	"	10.0	10.0-15.9			16.0-20.9			21.0-27.9			28.0 " "	
60	"	"	14.2	14.2-22.7			22.8-29.9			30.0-39.9			40.0 " "	
70	"	"	16.5	16.5-26.5			26.6-34.8			34.9-46.5			46.6 " "	
80	"	"	18.0	18.0-28.9			29.0-38.0			38.1-50.7			50.8 " "	

^{1/}Volume index or vi equals the basal area of the average dominant and co-dominant balsam fir tree times its height. The vi values given above are for stands with competition index 0.8. For other competition index classes use corrected vi (divide vi by factors given in table 6).

^{2/}This is the average age of the dominant and codominant trees in the main stand.

Table 6.--Correction factors to be applied to volume index (first estimate) of average dominant and codominant trees, upland balsam fir type

Competition index	:	Correction factor ^{1/}
.2		1.26
.4		1.17
.6		1.09
.8		1.00
1.0		.91
1.2		.84
1.4		.79
1.6		.75

^{1/} The product of the basal area of the average dominant and codominant balsam fir tree and its height (vi) divided by the correction factor equals vi adjusted to competition index 0.8.

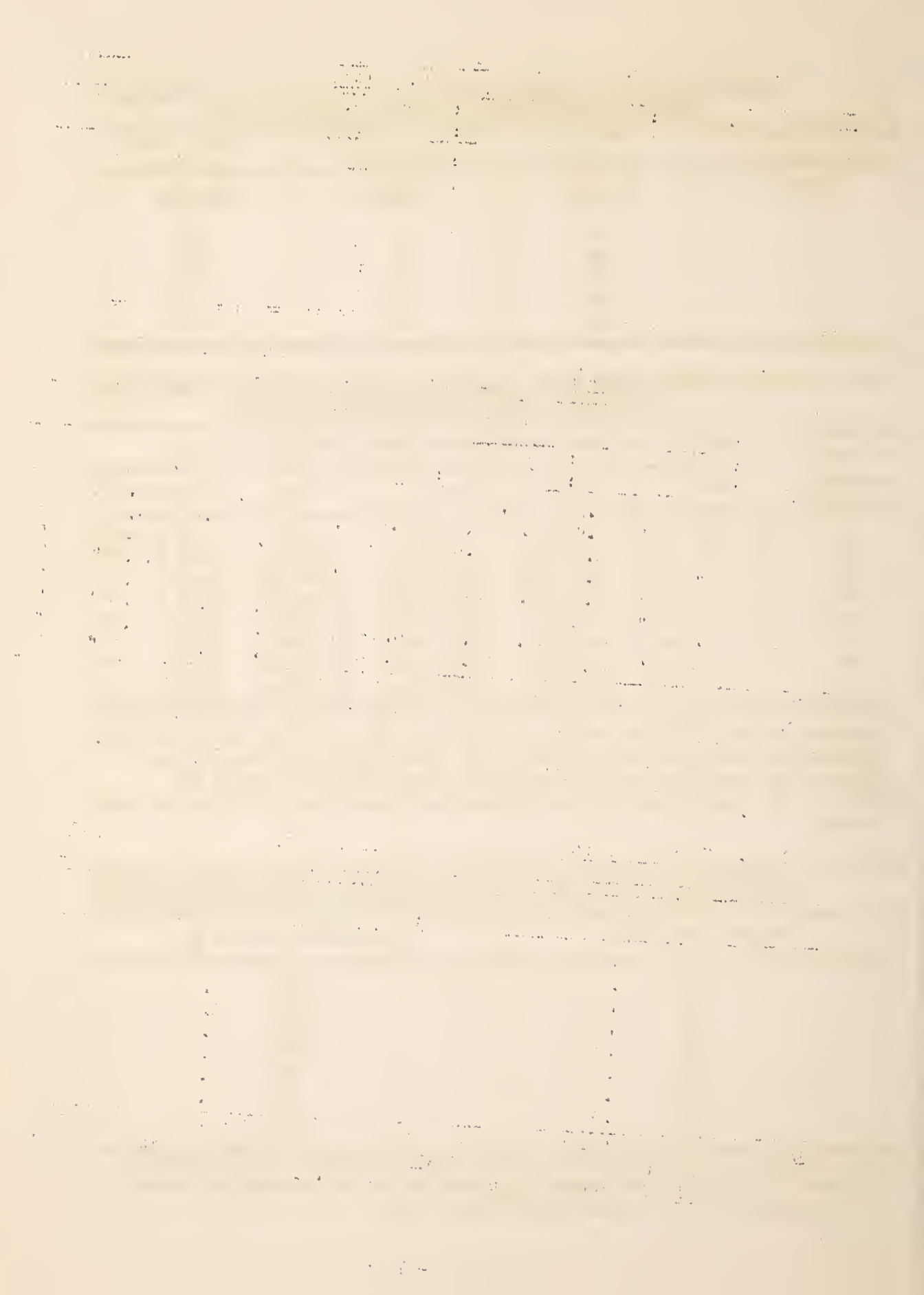


Table 7.--Expected growth per acre during the next 10 years in cubic feet in-
side bark (entire stem and stump), upland balsam fir type 1/

POOR SITE								
Stand age at breast height ^{2/} (years)	Ten years' growth in cubic feet when competition index is:							
	.4	.6	.8	1.0	1.2	1.4	1.6	
10	390	490	550	590	610	620	620	
20	440	550	620	660	690	700	710	
30	410	540	610	650	680	700	700	
40	290	370	430	480	500	510	520	
50	180	210	250	280	300	300	310	
60	60	70	70	80	80	90	100	
70	-60	-70	-80	-90	-110	-120	-130	
80	-150	-200	-230	-280	-320	-330	-330	
FAIR SITE								
10	470	580	650	710	750	760	770	
20	520	640	730	780	830	850	870	
30	450	560	620	660	700	710	720	
40	320	400	450	490	520	530	540	
50	200	240	270	300	320	320	320	
60	80	90	90	100	100	100	100	
70	-30	-50	-60	-70	-70	-80	-80	
80	-90	-130	-170	-190	-210	-210	-220	
MEDIUM SITE								
10	560	690	780	840	890	910	920	
20	580	740	830	890	920	940	950	
30	500	610	680	740	780	790	790	
40	370	430	490	540	550	560	560	
50	220	260	290	320	340	340	330	
60	80	100	110	120	120	130	140	
70	-20	-20	-30	-30	-30	-40	-40	
80	-50	-70	-90	-100	-110	-110	-110	
GOOD SITE								
10	650	800	900	970	1,040	1,030	1,050	
20	680	840	950	1,010	1,050	1,070	1,080	
30	530	660	740	800	830	840	840	
40	400	470	520	580	620	630	640	
50	250	290	330	360	390	400	400	
60	120	130	150	160	160	170	170	
70	10	20	20	20	20	20	20	
80	-40	-30	-50	-60	-60	-70	-70	
EXCELLENT SITE								
10	740	910	1,030	1,120	1,180	1,200	1,210	
20	760	950	1,080	1,140	1,180	1,200	1,210	
30	620	730	830	900	950	960	960	
40	450	530	600	660	690	700	700	
50	290	350	400	440	450	450	450	
60	150	170	200	220	230	230	230	
70	20	30	50	60	60	60	60	
80	-10	-20	-30	-40	-40	-40	-40	

1/ If stand is not 100 percent balsam fir, reduce table values proportionately

2/ This is the average age of the dominant and codominant trees in the main stand.

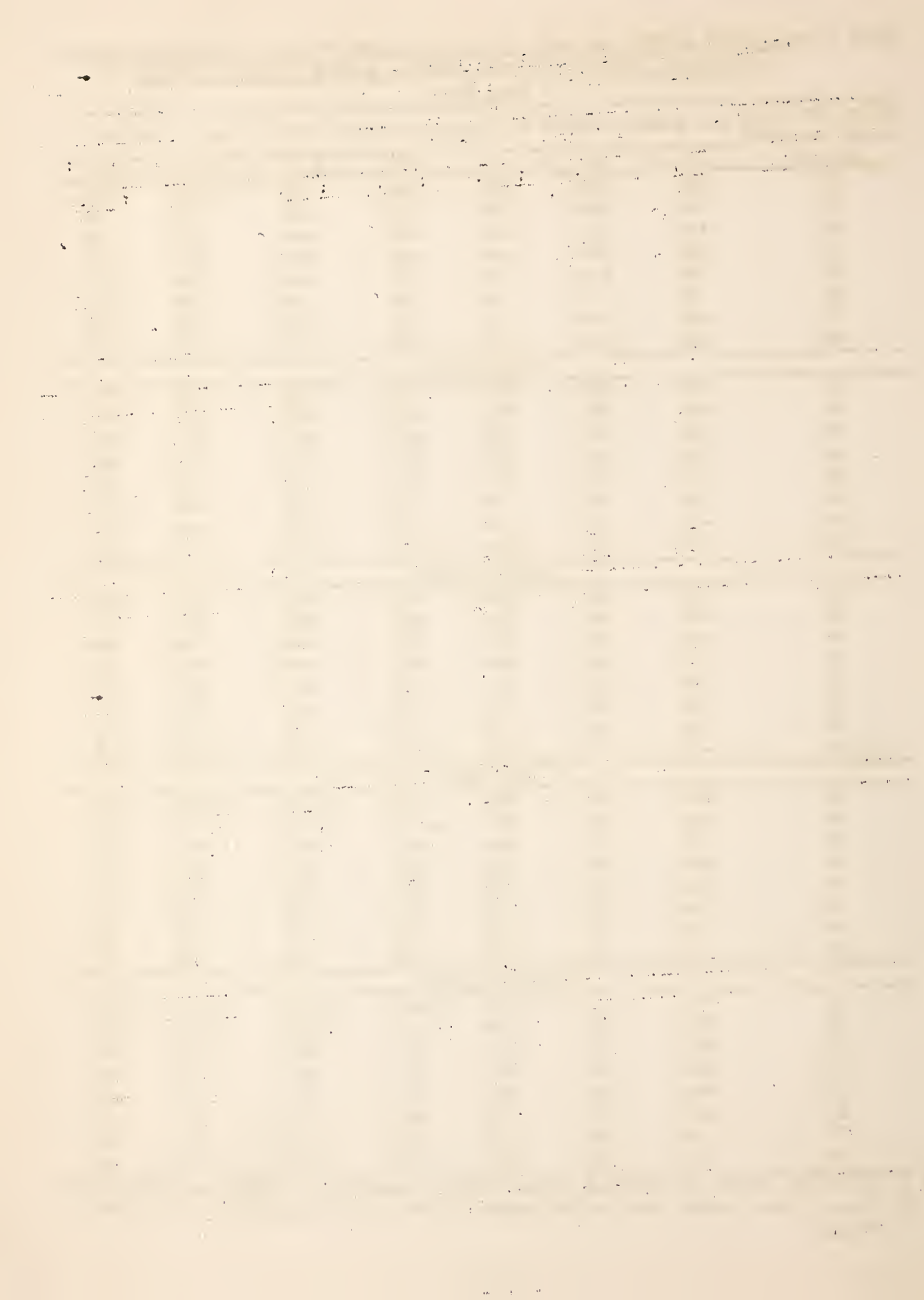


Table 8.--Expected growth per acre during the next 20 years, in cubic feet inside bark (entire stem and stump), upland balsam fir type 1/

POOR SITE

Stand age at breast height ^{2/} (years)	Twenty years' growth in cubic feet when competition index is:						
	.4	.6	.8	1.0	1.2	1.4	1.6
10	920	1,070	1,180	1,250	1,300	1,320	1,310
20	910	1,110	1,230	1,320	1,370	1,390	1,380
30	710	880	1,020	1,110	1,180	1,200	1,180
40	470	570	670	740	800	810	800
50	230	280	320	350	380	380	360
60	0	-10	-20	-30	-30	-40	-60
70	-210	-280	-340	-380	-430	-460	-480

FAIR SITE

10	1,090	1,270	1,420	1,520	1,580	1,600	1,600
20	1,040	1,210	1,360	1,460	1,530	1,540	1,550
30	830	960	1,070	1,160	1,220	1,250	1,240
40	540	640	720	790	840	850	830
50	270	310	360	390	420	440	430
60	40	40	40	40	30	20	0
70	-140	-180	-220	-260	-280	-310	-330

MEDIUM SITE

10	1,260	1,480	1,640	1,740	1,810	1,830	1,830
20	1,150	1,370	1,520	1,630	1,700	1,720	1,700
30	890	1,040	1,160	1,260	1,330	1,330	1,290
40	600	680	760	830	890	890	850
50	300	340	380	420	460	460	450
60	70	80	80	90	90	80	40
70	-60	-110	-130	-140	-140	-150	-190

GOOD SITE

10	1,460	1,700	1,880	2,000	2,060	2,070	2,080
20	1,280	1,530	1,700	1,810	1,880	1,900	1,860
30	960	1,100	1,240	1,360	1,450	1,440	1,350
40	730	800	880	950	1,010	1,010	980
50	370	420	460	510	550	570	550
60	140	150	160	170	180	190	200
70	-20	-20	-20	-30	-40	-70	-120

EXCELLENT SITE

10	1,660	1,940	2,140	2,260	2,360	2,390	2,390
20	1,460	1,670	1,870	2,030	2,130	2,150	2,110
30	1,110	1,270	1,410	1,540	1,640	1,650	1,620
40	760	880	980	1,070	1,140	1,150	1,110
50	440	510	580	640	680	690	670
60	250	260	270	280	290	280	280
70	-10	0	20	40	50	50	30

1/ If stand is not 100 percent balsam fir, reduce table values proportionately.

2/ This is the average age of the dominant and codominant trees in the main stand.

Table 9.--Expected growth per acre during the next 10 years, in rough cords, to a varying top diameter inside bark of not less than 3 inches, upland balsam fir type 1/

FOOR SITE							
Stand age at breast height 2/ (years)	: Ten years' growth in rough cords when competition index is:						
	: .4	: .6	: .8	: 1.0	: 1.2	: 1.4	: 1.6
10	.8	.9	.8	.7	.6	.6	.6
20	2.3	2.5	2.5	2.4	2.2	2.2	2.1
30	4.4	4.8	4.8	4.7	4.5	4.4	4.2
40	4.2	5.1	5.9	6.6	7.1	7.1	6.6
50	2.9	3.5	4.0	4.7	5.4	5.9	6.3
60	1.1	1.1	1.4	2.0	2.8	3.4	4.1
70	.1	.2	.2	.4	.7	1.2	1.7
80	-.9	-1.2	-1.5	-1.7	-1.6	-.9	-.3
FAIR SITE							
10	1.7	1.9	2.0	1.9	1.8	1.8	1.7
20	4.5	5.3	5.6	5.6	5.4	5.3	5.2
30	5.3	7.0	8.2	8.9	9.4	9.7	9.6
40	4.5	5.4	6.3	7.1	8.0	8.8	9.6
50	3.1	3.7	4.4	5.0	5.4	6.0	6.9
60	1.5	1.7	2.1	2.3	2.9	3.4	4.2
70	.2	.3	.5	.7	1.1	1.4	2.0
80	-.6	-.8	-.8	-.8	-.8	-.6	.1
MEDIUM SITE							
10	2.8	3.0	3.1	3.2	3.2	3.2	3.0
20	6.0	7.4	8.2	8.4	8.3	8.2	8.0
30	6.2	7.8	9.1	10.2	11.2	12.0	12.6
40	4.9	6.0	6.9	7.9	8.8	9.4	9.9
50	3.3	4.0	4.6	5.2	5.8	6.3	6.9
60	1.7	1.9	2.2	2.5	3.2	3.9	4.3
70	.4	.6	.7	.9	1.2	1.6	2.1
80	-.3	-.4	-.5	-.5	-.4	-.2	.1
GOOD SITE							
10	4.6	5.1	5.4	5.5	5.3	5.2	4.9
20	7.3	8.8	10.1	11.1	11.7	12.0	12.1
30	6.6	8.4	9.8	11.0	12.2	13.2	14.4
40	5.4	6.5	7.5	8.6	9.6	10.0	10.7
50	3.5	4.3	5.0	5.6	6.1	6.6	7.1
60	1.9	2.1	2.5	2.9	3.5	4.0	4.7
70	.6	.8	1.0	1.2	1.5	1.8	2.1
80	0	0	0	.1	.2	.4	.7
EXCELLENT SITE							
10	6.2	7.0	7.2	7.2	7.1	7.1	6.9
20	8.5	10.6	12.2	13.2	14.1	14.6	14.8
30	7.9	9.8	11.2	12.2	13.2	14.2	15.3
40	6.0	7.4	8.7	9.8	10.6	11.1	11.6
50	3.8	4.8	5.6	6.3	6.9	7.3	8.2
60	2.0	2.4	2.9	3.4	3.8	4.3	5.3
70	.8	1.1	1.4	1.6	1.9	2.2	2.5
80	.2	.3	.4	.6	.7	.8	1.0

1/ If stand is not 100 percent balsam fir, reduce table values proportionately.

2/ This is the average age of the dominant and codominant trees in the main stand.

Table 10.--Expected growth per acre during the next 20 years, in rough cords, to a varying top diameter inside bark of not less than 3 inches, upland balsam fir type 1/

POOR SITE							
Stand age at breast height ^{2/} (years)	Twenty years' growth in rough cords when competition index is:						
	.4	.6	.8	1.0	1.2	1.4	1.6
10	3.3	3.5	3.4	3.1	2.8	2.8	2.8
20	7.2	7.3	7.2	7.0	6.7	6.6	6.5
30	8.7	10.0	10.8	11.3	11.6	11.6	11.3
40	7.3	8.4	9.9	11.7	12.5	12.9	12.8
50	4.1	4.5	5.5	6.9	8.2	9.3	10.2
60	1.3	1.4	1.8	2.4	3.3	4.5	5.4
70	-.8	-1.0	-1.2	-1.2	-.9	-.1	1.1

FAIR SITE							
10	7.0	7.4	7.5	7.4	7.2	7.2	7.1
20	10.9	12.6	13.9	14.6	14.8	14.9	14.9
30	10.1	12.5	14.5	16.2	17.4	18.1	18.7
40	7.8	9.2	10.6	12.1	13.4	14.7	15.9
50	4.6	5.2	6.0	7.1	8.3	9.6	10.7
60	1.6	1.8	2.2	3.0	4.0	5.0	5.8
70	-.4	-.5	-.4	-.1	.3	.7	1.2

MEDIUM SITE							
10	9.9	11.0	11.6	11.6	11.5	11.5	11.3
20	13.1	15.8	17.7	18.8	19.5	20.4	21.4
30	11.5	13.8	16.1	18.2	20.0	21.2	21.9
40	8.3	10.0	11.7	13.2	14.6	15.6	16.3
50	5.0	5.4	6.4	7.7	9.0	10.1	10.9
60	2.0	2.1	2.7	3.4	4.4	5.2	5.9
70	.1	0	0	.3	.8	1.2	1.7

GOOD SITE							
10	13.1	14.8	16.0	16.8	17.0	17.0	16.8
20	14.9	17.8	20.1	22.0	23.4	24.4	25.0
30	12.5	14.9	17.2	19.4	21.3	22.8	24.0
40	8.8	10.6	12.4	14.1	15.7	16.6	17.0
50	5.5	6.4	7.3	8.3	9.4	10.4	11.4
60	2.6	2.9	3.4	4.1	4.9	5.7	6.1
70	.6	.7	1.0	1.3	1.8	2.1	2.4

EXCELLENT SITE							
10	16.6	18.5	19.8	20.6	21.2	21.3	21.3
20	17.3	20.9	23.6	25.6	27.3	28.4	29.1
30	14.4	17.2	19.7	22.0	23.8	25.1	26.2
40	9.3	11.6	14.0	15.9	17.3	18.3	18.9
50	5.9	7.1	8.3	9.4	10.5	11.5	12.4
60	2.9	3.6	4.3	5.0	5.7	6.3	6.9
70	1.2	1.4	1.8	2.2	2.6	2.9	3.3

1/If stand is not 100 percent balsam fir, reduce table values proportionately.

2/This is the average age of the dominant and codominant trees in the main stand.

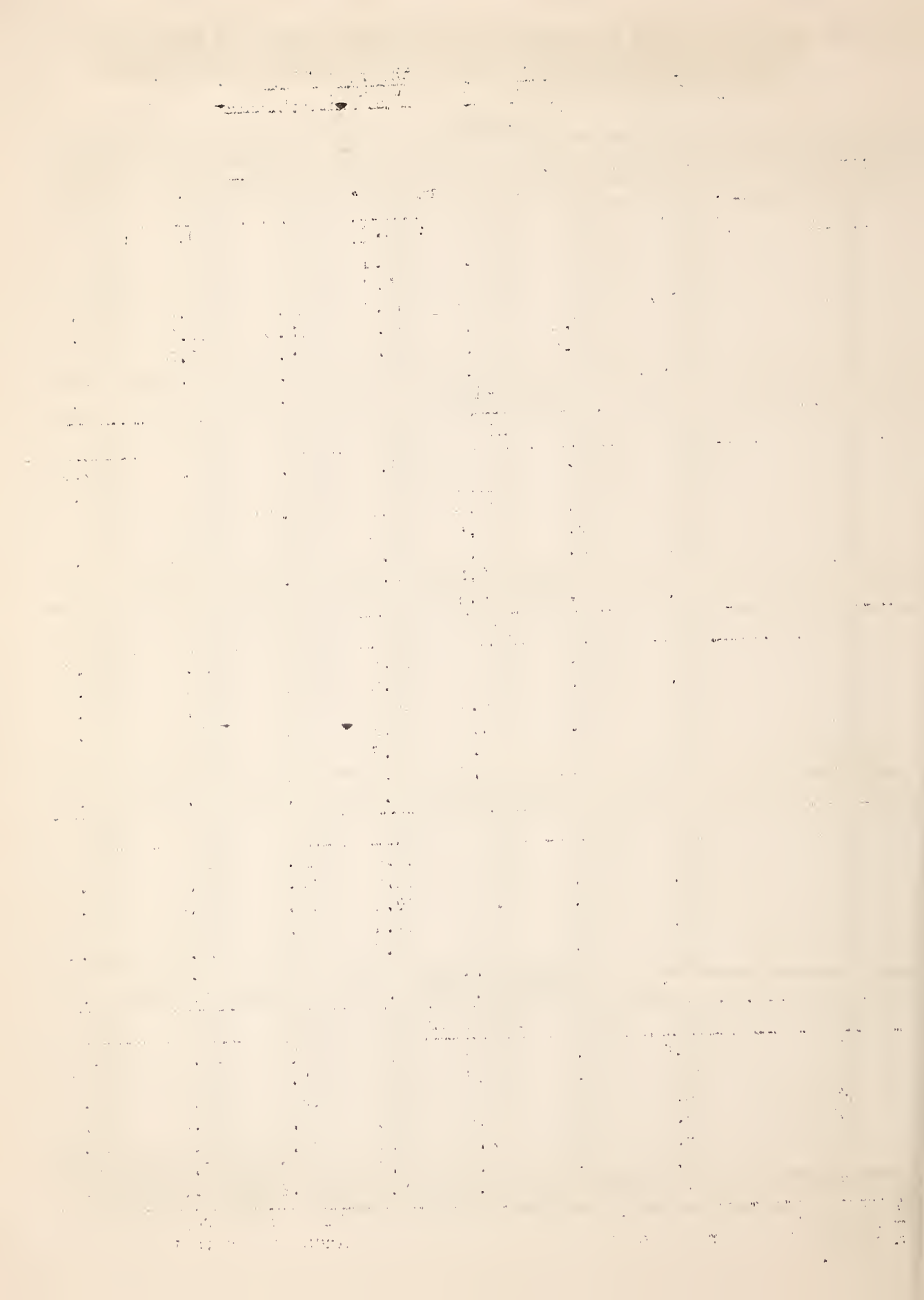


Table 11.--Expected growth per acre during the next 10 years, in rough cords, to a varying top diameter inside bark of not less than 4 inches, upland balsam fir type 1/

POOR SITE								
Stand age at breast height ^{2/} (years)	: Ten years' growth in rough cords when competition index is:							
	: .4	: .6	: .8	: 1.0	: 1.2	: 1.4	: 1.6	
10	.3	.2	.2	.1	0	0	0	
20	1.4	1.3	1.2	1.1	.9	.8	.6	
30	2.9	2.9	2.9	2.9	2.9	2.9	2.8	
40	3.7	4.4	4.8	4.8	4.6	4.5	4.4	
50	2.7	3.3	3.9	4.5	4.9	5.3	5.4	
60	1.6	1.8	2.2	2.9	3.7	4.4	5.0	
70	.4	.6	.9	1.2	1.5	2.1	2.8	
80	-.5	-.6	-.6	-.6	-.5	0	.7	
FAIR SITE								
10	.9	.9	.9	.8	.7	.7	.6	
20	3.3	3.5	3.4	3.3	3.0	2.9	2.8	
30	4.9	6.1	6.9	7.3	7.3	7.1	6.7	
40	4.3	5.2	6.1	7.1	8.0	8.7	9.1	
50	2.9	3.5	4.1	4.8	5.7	6.5	7.4	
60	1.7	2.0	2.4	3.0	3.7	4.4	5.2	
70	.4	.7	1.0	1.3	1.8	2.4	3.0	
80	-.5	-.5	-.4	-.3	-.1	.2	.9	
MEDIUM SITE								
10	1.8	1.9	1.8	1.7	1.6	1.4	1.3	
20	4.9	5.6	5.9	6.0	5.8	5.7	5.6	
30	6.3	8.0	9.1	9.6	10.0	10.6	11.2	
40	4.8	5.9	6.9	7.9	8.8	9.5	10.1	
50	3.2	3.9	4.5	5.2	6.0	6.7	7.4	
60	1.8	2.1	2.6	3.1	3.9	4.6	5.4	
70	.6	.9	1.2	1.5	1.9	2.4	3.3	
80	-.3	-.3	-.2	-.2	-.2	.6	1.2	
GOOD SITE								
10	3.2	3.5	3.4	3.1	2.8	2.7	2.5	
20	6.6	7.6	8.4	8.8	8.9	8.9	8.8	
30	7.0	8.4	9.8	10.7	12.0	12.6	12.9	
40	5.2	6.1	7.1	8.1	9.1	10.0	10.9	
50	3.5	4.4	5.2	6.0	6.7	7.3	7.7	
60	1.9	2.2	2.7	3.2	4.0	4.7	5.4	
70	.7	1.0	1.4	1.7	2.2	2.8	3.5	
80	0	0	.2	.4	.6	1.0	1.5	
EXCELLENT SITE								
10	4.5	4.8	4.8	4.7	4.4	4.2	4.0	
20	8.0	9.6	10.8	11.7	12.3	12.5	12.5	
30	7.6	9.5	11.1	12.5	13.7	14.9	16.0	
40	5.8	7.1	8.2	9.1	9.8	10.7	11.4	
50	3.9	4.9	5.8	6.6	7.3	7.9	8.5	
60	2.2	2.7	3.2	3.6	4.1	4.9	6.1	
70	.9	1.4	1.8	2.2	2.7	3.1	3.6	
80	.2	.4	.6	.8	1.0	1.2	1.6	

1/ If stand is not 100 percent balsam fir, reduce table values proportionately.

2/ This is the average age of the dominant and codominant trees in the main stand.

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[illegible]

Table 12.--Expected growth per acre during the next 20 years, in rough cords, to a varying top diameter inside bark of not less than 4 inches, upland balsam fir type 1/

POOR SITE							
Stand age at breast height ^{2/} (years)	Twenty years' growth in rough cords when competition index is:						
	.4	.6	.8	1.0	1.2	1.4	1.6
10	1.6	1.5	1.4	1.1	.9	.8	.7
20	4.2	4.2	4.2	4.0	3.8	3.6	3.5
30	6.9	7.5	7.7	7.8	7.5	7.4	7.3
40	6.6	7.5	8.5	9.2	9.5	9.8	9.9
50	4.3	4.9	5.9	7.4	8.6	9.4	10.0
60	2.1	2.5	3.2	4.2	5.2	6.3	7.5
70	0	.1	.4	.7	1.0	1.6	3.2
FAIR SITE							
10	4.3	4.3	4.2	4.0	3.7	3.6	3.5
20	9.0	10.1	10.6	10.6	10.3	10.1	10.0
30	9.8	11.6	13.2	14.4	15.3	15.7	15.6
40	7.4	8.8	10.4	12.0	13.7	15.0	16.0
50	4.7	5.4	6.6	7.9	9.4	10.8	12.1
60	2.2	2.6	3.4	4.4	5.5	6.6	7.8
70	0	.2	.6	1.0	1.7	2.7	3.7
MEDIUM SITE							
10	7.2	7.7	7.8	7.7	7.4	7.2	7.1
20	12.0	13.9	15.1	15.7	15.8	15.9	16.1
30	11.4	13.7	15.8	17.5	18.8	19.8	20.7
40	8.1	9.7	11.4	13.2	14.8	16.0	17.1
50	5.1	5.9	7.1	8.5	9.9	11.2	12.2
60	2.4	2.9	3.8	4.7	5.9	7.0	8.3
70	.3	.8	1.2	1.6	2.1	2.8	3.9
GOOD SITE							
10	10.6	11.6	12.0	11.9	11.7	11.6	11.5
20	14.5	16.7	18.6	20.2	21.3	21.8	21.8
30	12.5	15.1	17.5	19.7	21.5	22.7	24.0
40	8.6	10.5	12.4	14.1	15.8	17.4	18.8
50	5.4	6.5	7.8	9.2	10.7	11.9	13.0
60	2.6	3.4	4.3	5.2	6.2	7.4	8.7
70	.7	1.1	1.6	2.1	2.8	3.5	4.4
EXCELLENT SITE							
10	13.9	15.4	16.3	16.6	16.7	16.6	16.5
20	16.9	20.0	22.4	24.4	26.0	27.0	27.5
30	13.7	16.5	19.1	21.5	23.5	25.3	26.8
40	9.3	11.8	13.6	15.5	17.3	18.7	19.7
50	6.2	7.6	9.0	10.3	11.5	12.5	13.7
60	2.9	3.8	4.8	5.7	6.8	7.8	9.0
70	.9	1.7	2.4	3.0	3.7	4.2	4.7

1/ If stand is not 100 percent balsam fir, reduce table values proportionately.

2/ This is the average age of the dominant and codominant trees in the main stand.

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DEPARTMENT OF PHYSICS
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Table 13.--Gross volume per acre in cubic feet inside bark (including stem and stump), upland balsam fir type 1/

POOR SITE								
Stand age at breast height ^{2/} :	Cubic-foot volume per acre when competition index is:							
	.4	.6	.8	1.0	1.2	1.4	1.6	
Years								
10	150	190	240	280	310	320	330	
20	440	570	710	830	920	960	980	
30	810	1,000	1,240	1,450	1,610	1,680	1,700	
40	1,110	1,420	1,770	2,070	2,290	2,390	2,400	
50	1,320	1,720	2,140	2,530	2,790	2,900	2,930	
60	1,460	1,890	2,360	2,790	3,090	3,200	3,230	
70	1,500	1,930	2,420	2,850	3,170	3,300	3,320	
80	1,440	1,860	2,320	2,720	3,060	3,170	3,190	
90	1,290	1,650	2,100	2,440	2,740	2,860	2,850	
FAIR SITE								
10	170	210	260	310	340	360	360	
20	520	680	830	980	1,090	1,130	1,140	
30	910	1,190	1,450	1,720	1,920	1,980	2,000	
40	1,250	1,640	2,010	2,360	2,620	2,720	2,750	
50	1,490	1,950	2,400	2,840	3,140	3,250	3,300	
60	1,640	2,140	2,640	3,120	3,460	3,580	3,630	
70	1,690	2,180	2,720	3,210	3,560	3,710	3,740	
80	1,660	2,130	2,650	3,130	3,490	3,640	3,650	
90	1,550	2,030	2,500	2,940	3,280	3,430	3,430	
MEDIUM SITE								
10	180	230	280	340	370	390	390	
20	600	780	960	1,140	1,260	1,300	1,330	
30	1,030	1,360	1,680	1,980	2,180	2,280	2,300	
40	1,400	1,820	2,270	2,670	2,960	3,080	3,110	
50	1,670	2,170	2,700	3,170	3,510	3,630	3,690	
60	1,830	2,380	2,970	3,470	3,850	3,980	4,050	
70	1,890	2,450	3,030	3,550	3,970	4,130	4,190	
80	1,880	2,420	3,050	3,510	3,940	4,100	4,150	
90	1,830	2,350	2,950	3,420	3,830	3,980	4,040	
GOOD SITE								
10	200	260	320	380	420	440	440	
20	680	890	1,100	1,300	1,430	1,500	1,520	
30	1,190	1,550	1,910	2,270	2,480	2,600	2,650	
40	1,580	2,060	2,530	3,010	3,310	3,430	3,490	
50	1,870	2,430	2,990	3,560	3,930	4,050	4,110	
60	2,050	2,680	3,280	3,920	4,320	4,470	4,510	
70	2,130	2,770	3,430	4,050	4,480	4,670	4,700	
80	2,140	2,780	3,460	4,070	4,500	4,720	4,720	
90	2,120	2,750	3,400	4,010	4,440	4,650	4,650	
EXCELLENT SITE								
10	220	290	360	430	470	490	490	
20	790	1,030	1,280	1,480	1,650	1,710	1,740	
30	1,360	1,760	2,200	2,580	2,830	2,960	2,980	
40	1,820	2,350	2,900	3,410	3,780	3,910	3,980	
50	2,140	2,790	3,430	4,030	4,470	4,630	4,720	
60	2,360	3,080	3,790	4,470	4,920	5,110	5,210	
70	2,470	3,220	3,980	4,680	5,150	5,390	5,430	
80	2,490	3,250	4,030	4,730	5,210	5,490	5,490	
90	2,480	3,230	4,000	4,720	5,200	5,450	5,470	

1/If stand is not 100 percent balsam fir, reduce table values proportionately.

2/This is the average age of the dominant and codominant trees in the main stand.

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and the role of the accounting department in ensuring the integrity of the financial data. It emphasizes the need for transparency and accountability in all financial reporting.

2. The second part of the document outlines the various methods used to collect and analyze financial data, including the use of spreadsheets, databases, and specialized accounting software. It also discusses the importance of regular audits and the role of external auditors in verifying the accuracy of the financial statements.

3. The third part of the document focuses on the preparation and presentation of financial statements, including the balance sheet, income statement, and cash flow statement. It provides detailed instructions on how to format these statements and how to interpret the results.

4. The fourth part of the document discusses the importance of maintaining accurate records of all transactions and the role of the accounting department in ensuring the integrity of the financial data. It emphasizes the need for transparency and accountability in all financial reporting.

5. The fifth part of the document outlines the various methods used to collect and analyze financial data, including the use of spreadsheets, databases, and specialized accounting software. It also discusses the importance of regular audits and the role of external auditors in verifying the accuracy of the financial statements.

6. The sixth part of the document focuses on the preparation and presentation of financial statements, including the balance sheet, income statement, and cash flow statement. It provides detailed instructions on how to format these statements and how to interpret the results.

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8. The eighth part of the document outlines the various methods used to collect and analyze financial data, including the use of spreadsheets, databases, and specialized accounting software. It also discusses the importance of regular audits and the role of external auditors in verifying the accuracy of the financial statements.

9. The ninth part of the document focuses on the preparation and presentation of financial statements, including the balance sheet, income statement, and cash flow statement. It provides detailed instructions on how to format these statements and how to interpret the results.

10. The tenth part of the document discusses the importance of maintaining accurate records of all transactions and the role of the accounting department in ensuring the integrity of the financial data. It emphasizes the need for transparency and accountability in all financial reporting.

Table 14.--Gross merchantable volume per acre, in rough cords, to a varying top diameter inside bark of not less than 3 inches, upland balsam fir type 1/

POOR SITE

Stand age at breast height $\frac{2}{3}$ (Years)	Rough cords per acre when competition index is:						
	.4	.6	.8	1.0	1.2	1.4	1.6
10	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)	0	0
20	1	1	1	1	1	1	1
30	3	3	4	3	3	3	3
40	6	8	8	8	7	7	6
50	10	13	14	15	14	13	12
60	13	16	18	20	20	19	17
70	14	17	19	22	23	22	21
80	14	17	20	22	23	23	22
90	13	16	18	20	22	22	22

FAIR SITE

10	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)
20	2	2	2	2	2	2	2
30	6	7	8	8	7	7	6
40	10	13	15	16	17	16	14
50	14	18	21	24	25	24	23
60	17	21	25	29	30	30	29
70	18	23	27	31	33	34	33
80	18	23	28	32	34	35	34
90	18	22	27	31	33	34	34

MEDIUM SITE

10	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)	($\frac{3}{3}$)
20	3	3	3	4	4	3	3
30	8	10	11	12	12	11	10
40	13	16	20	22	23	23	21
50	17	22	26	30	32	32	30
60	20	25	30	35	38	38	37
70	21	27	32	38	41	41	41
80	21	27	33	39	42	43	42
90	21	27	33	38	42	43	42

GOOD SITE

10	1	1	1	1	1	1	($\frac{3}{3}$)
20	4	5	6	6	6	5	4
30	11	13	15	17	18	16	14
40	16	20	24	28	29	29	28
50	20	26	31	36	39	39	38
60	23	29	36	41	45	45	45
70	25	31	38	44	48	49	49
80	26	32	39	45	50	51	50
90	26	32	39	45	50	51	50

EXCELLENT SITE

10	1	1	1	1	1	1	1
20	6	7	8	8	8	7	6
30	13	16	19	21	22	21	20
40	19	24	30	33	35	35	35
50	24	31	37	43	46	46	46
60	27	35	42	49	52	54	54
70	28	37	45	52	56	58	58
80	29	38	46	54	58	60	60
90	29	38	47	54	59	61	61

1/ If stand is not 100 percent balsam fir, reduce table values proportionately.

2/ This is the average age of the dominant and codominant trees in the main stand. 3/ less than 0.5 cord.

Table 15.--Gross merchantable volume per acre, in rough cords, to a varying top diameter inside bark of not less than 4 inches, upland balsam fir type 1/

Stand age at breast height ^{2/} : (years)	FOOR SITE						
	Rough cords per acre when competition index is:						
	.4	.6	.8	1.0	1.2	1.4	1.6
10	0	0	0	0	0	0	0
20	(3/)	(3/)	(3/)	(3/)	0	0	0
30	1	2	2	1	1	1	1
40	4	4	4	4	4	3	3
50	8	9	9	9	8	7	6
60	10	12	13	14	13	12	10
70	12	14	15	16	17	16	14
80	12	14	16	18	18	18	16
90	12	14	16	17	18	18	16
	FAIR SITE						
	.4	.6	.8	1.0	1.2	1.4	1.6
10	0	0	0	0	0	0	0
20	1	1	1	1	1	1	(3/)
30	4	4	4	4	4	3	2
40	9	10	11	12	11	10	8
50	12	15	17	19	19	18	16
60	15	18	21	23	25	24	22
70	16	20	24	26	28	28	26
80	17	21	25	28	30	30	29
90	16	20	24	28	30	30	29
	MEDIUM SITE						
	.4	.6	.8	1.0	1.2	1.4	1.6
10	(3/)	(3/)	(3/)	0	0	0	0
20	2	2	2	2	2	1	1
30	6	7	8	8	7	6	5
40	12	14	16	17	17	16	15
50	15	19	23	25	26	25	24
60	18	23	27	30	32	32	30
70	20	25	30	33	36	36	35
80	20	26	31	35	38	39	38
90	20	26	31	35	38	39	38
	GOOD SITE						
	.4	.6	.8	1.0	1.2	1.4	1.6
10	(3/)	(3/)	(3/)	(3/)	(3/)	(3/)	0
20	3	4	4	3	3	2	2
30	9	10	12	12	12	11	9
40	15	18	21	23	24	23	21
50	19	24	28	32	33	33	32
60	21	27	33	38	40	40	39
70	23	29	36	41	44	45	44
80	24	30	37	43	46	48	47
90	24	30	37	43	47	48	47
	EXCELLENT SITE						
	.4	.6	.8	1.0	1.2	1.4	1.6
10	(3/)	(3/)	(3/)	(3/)	(3/)	(3/)	(3/)
20	4	5	5	5	5	4	3
30	11	14	16	17	17	16	14
40	17	22	26	29	31	30	29
50	22	28	33	38	40	41	40
60	25	32	39	45	48	48	47
70	27	35	42	48	52	53	53
80	28	36	44	50	55	56	56
90	28	36	44	51	56	57	57

1/ If stand is not 100 percent balsam fir, reduce table values proportionately.

2/ This is the average age of the dominant and codominant trees in the main stand.

3/ less than 0.5 cord.

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and the role of the accounting department in ensuring the integrity of the financial statements. It also highlights the need for regular audits and the importance of transparency in financial reporting.

2. The second part of the document focuses on the implementation of internal controls to prevent fraud and ensure the accuracy of financial data. It outlines the key components of a robust internal control system, including segregation of duties, authorization procedures, and regular monitoring and evaluation.

3. The third part of the document addresses the challenges faced by organizations in managing their financial resources effectively. It discusses the importance of budgeting, forecasting, and cost management, and provides practical advice on how to overcome common financial management challenges.

4. The fourth part of the document explores the role of technology in modern accounting and finance. It discusses the benefits of using accounting software and the importance of staying up-to-date with the latest technological advancements in the field.

5. The fifth part of the document provides a summary of the key points discussed and offers final thoughts on the importance of financial management for the success of any organization. It emphasizes the need for a proactive approach to financial management and the importance of continuous learning and improvement.

Table 16.--Cumulative table of basal area per acre for tree tallies on 1/10-acre plots only

D.b.h. class		: Basal area per acre in square feet for the following number of trees listed on 1/10-acre plot--										
		:Tens	: 0	: 1	: 2	: 3	: 4	: 5	: 6	: 7	: 8	: 9
Inches		Square feet										
1	0	..	.05	.10	.15	.20	.25	.30	.35	.40	.45	
	10+	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95	
	20+	1.00	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	
	30+	1.50	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	
	40+	2.00	2.05	2.10	2.15	2.20	2.25	2.30	2.35	2.40	2.45	
	50+	2.50	2.55	2.60	2.65	2.70	2.75	2.80	2.85	2.90	2.95	
	60+	3.00	3.05	3.10	3.15	3.20	3.25	3.30	3.35	3.40	3.45	
2	0	..	.22	.44	.66	.88	1.10	1.32	1.54	1.76	1.98	
	10+	2.20	2.42	2.64	2.86	3.08	3.30	3.52	3.74	3.96	4.18	
	20+	4.40	4.62	4.84	5.06	5.28	5.50	5.72	5.94	6.16	6.38	
	30+	6.60	6.82	7.04	7.26	7.48	7.70	7.92	8.14	8.36	8.58	
	40+	8.80	9.02	9.24	9.46	9.68	9.90	10.12	10.34	10.56	10.78	
	50+	11.00	11.22	11.44	11.66	11.88	12.10	12.32	12.54	12.76	12.98	
3	0	..	.49	.98	1.47	1.96	2.45	2.94	3.43	3.92	4.41	
	10+	4.90	5.39	5.88	6.37	6.86	7.35	7.84	8.33	8.82	9.31	
	20+	9.80	10.29	10.78	11.27	11.76	12.25	12.74	13.23	13.72	14.21	
	30+	14.70	15.19	15.68	16.17	16.66	17.15	17.64	18.13	18.62	19.11	
	40+	19.60	20.09	20.58	21.07	21.56	22.05	22.54	23.03	23.52	24.01	
4	0	..	.87	1.74	2.61	3.48	4.35	5.22	6.09	6.96	7.83	
	10+	8.70	9.57	10.44	11.31	12.18	13.05	13.92	14.79	15.66	16.53	
	20+	17.40	18.27	19.14	20.01	20.88	21.75	22.62	23.49	24.36	25.23	
	30+	26.10	26.97	27.84	28.71	29.58	30.45	31.32	32.19	33.06	33.93	
	40+	34.80	35.67	36.54	37.41	38.28	39.15	40.02	40.89	41.76	42.63	
5	0	..	1.36	2.72	4.08	5.44	6.80	8.16	9.52	10.88	12.24	
	10+	13.60	14.96	16.32	17.68	19.04	20.40	21.76	23.12	24.48	25.84	
	20+	27.20	28.56	29.92	31.28	32.64	34.00	35.36	36.72	38.08	39.44	
	30+	40.80	42.16	43.52	44.88	46.24	47.60	48.96	50.32	51.68	53.04	
6	0	..	1.96	3.92	5.88	7.84	9.80	11.76	13.72	15.68	17.64	
	10+	19.60	21.56	23.52	25.48	27.44	29.40	31.36	33.32	35.28	37.24	
	20+	39.20	41.16	43.12	45.08	47.04	49.00	50.96	52.92	54.88	56.84	
7	0	..	2.67	5.34	8.01	10.68	13.35	16.02	18.69	21.36	24.03	
	10+	26.70	29.37	32.04	34.71	37.38	40.05	42.72	45.39	48.06	50.73	
8	0	..	3.49	6.98	10.47	13.96	17.45	20.94	24.43	27.92	31.41	
	10+	34.90	38.39	41.88	45.37	48.86	52.35	55.84	59.33	62.82	66.31	
9	0	..	4.42	8.84	13.26	17.68	22.10	26.52	30.94	35.36	39.78	
10	0	..	5.45	10.90	16.35	21.80	27.25	32.70	38.15	43.60	49.05	
11	0	..	6.60	13.20	19.80	26.40	33.00	39.60	46.20	52.80	59.40	
12	0	..	7.85	15.70	23.55	31.40	39.25	47.10	54.95	62.80	70.65	
13	0	..	9.22	18.44	27.66	36.88	46.10	55.32	64.54	73.76	82.98	
14	0	..	10.69	21.38	32.07	42.76	53.45	64.14	74.83	85.52	96.21	
15	0	..	12.27	24.54	36.81	49.08	61.35	73.62	85.89	98.16		
16	0	..	13.96	27.92	41.88	55.84						
17	0	..	15.76	31.52	47.28	63.04						
18	0	..	17.67	35.34								
19	0	..	19.69	39.38								
20	0	..	21.82	43.64								
21	0	..	24.05									
22	0	..	26.40									
23	0	..	28.85									

Example: If there are 56 two-inch trees on a 1/10-acre plot, the basal area per acre of 2-inch trees will be found in the sixth line under 2-inch d.b.h. as 12.32 square feet.

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